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A NEUTRAL-POINT EXPANSION OF THE IDEAL MAGNETOSPHERE

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Introduction.

In the idealized model of tne geomagnetic field - solar wind interaction (Beard, 1960) we assume the field to be excluded from the plasma and contained in a cavity called the magnetosphere. The boundary of the cavity, or magnetopause, is unknown, but we stipulate the dynamic condition that the magnetic pressure just inside the magnetopause is exactly balanced by the kinetic pressure of solar wind particles elastically reflected from the surface (thermal effects are neglected). Fig. 1 is a schematic drawing of the magnetopause, with the dipole located at the origin of coordinates, and perpendicular to the plasma stream-direction.

Mathematically, this situation is described by the field

$$\underline{H} = - \operatorname{grad} \Omega \ , \quad \nabla^2\Omega = 0 \ , \tag{1}$$

inside the magnetosphere, and the boundary conditions

$$\begin{aligned} &\underline{H}\cdot\operatorname{grad} F = 0 \ , \quad \text{(confinement)} \\ &\underline{H}\cdot\underline{H} = \beta^2\cos^2\chi \ , \ \text{(pressure)} \end{aligned} \tag{2}$$

which hold on the surface, $F(x',y',z') = 0$, where χ is the angle of incidence of the incoming plasma (Fig. 1), and $\beta^2 = 8\pi(2nmV^2)$. V, n, and m are the plasma drift-velocity, ion number density, and ion mass, respectively.

A singular point on this unknown surface is N, the neutral point, where the magnetic field lines "split," the field vanishes, $\chi = \pi/2$, and the gradient is in the x'-direction. Although the problem is unsolved as yet, there are some good approximations to the general shape (Mead and Beard, 1963; Spreiter and Briggs, 1962; Midgeley and Davis, 1963). However, these approximations generally fail in the region of the neutral point, a region of interest since, if it does exist, it is likely to be unstable, and a possible point of entry for high-energy particles into the magnetosphere.

The neutral point considered here is an X-type neutral point (Dungey, 1958, pp. 39-41,51-52,98-102); however, it lies <u>on</u> a bounding surface, which must run parallel to one of the limiting field lines (Fig. 2). We must assume that surface currents can be made to account for the disappearance of the field outside the surface (magnetopause).

<u>Series Representation</u>.

We can represent the field $\underline{H} = \beta \underline{h}$ by means of its scalar potential Ω. We first transform to the (x,y,z) coordinate system with origin at N such that

$$R(dx,dy,dz) = (dx',dy',dz') \; ; \qquad (3)$$

in a region small compared to the apex radius, R, we have $x^2 + y^2 + z^2 << 1$. The scalar potential is expanded in the form

$$-\Omega/\beta = ax^2 + by^2 + cz^2 + dx^3 + ey^3 + fz^3$$
$$+ mxy^2 + nxz^2 + rzx^2 + szy^2 \quad . \tag{4}$$

We must include cubic terms in the potential, since we expect the limiting field lines to be curvilinear in the xz-plane. There are no linear terms, because the field must vanish at N; terms in y to the first power are deleted, because symmetry requires $H_y = 0$ in the xz-plane. The dependence on xz has also been omitted, since for $y = 0 = z$ there should only be an x-component of H on the x-axis (Fig. 2).

The gradient of Eq. (4) yields the normalized field:

$$h_x = 2ax + 3dx^2 + my^2 + nz^2 + 2rxz;$$
$$h_y = 2by + 3ey^2 + 2mxy + 2szy;$$
$$h_z = 2cz + 3fz^2 + 2nxz + rx^2 + sy^2; \tag{5}$$

and the requirement $\underline{\nabla}\cdot\underline{H} = 0$ yields

$$a + b + c = 0 \;; \qquad 3d + m + n = 0 \;;$$
$$3f + r + s = 0 \;; \qquad e = 0 \;. \tag{6}$$

Furthermore, as indicated in Fig. 2, we expect $a < 0$, $c > 0$.

Noon Meridian Contour.

In the xz-plane the boundary conditions, Eq. (2), on the noon meridian contour become ($dx/dz = x'$):

$$h_x/h_z = - F_z/F_x = x' \; ; \tag{7}$$

$$h_x^2 + h_z^2 = x'^2/(1+x'^2). \tag{8}$$

If we substitute for x' in Eqs. (7), (8), then

$$h_x^4 + h_x^2(2h_z^2 - 1) + h_z^4 = 0 \; , \tag{9}$$

and since both components must vanish at N

$$h_x^2 = (1 - 2h_z^2 - (1-4h_z^2)^{\frac{1}{2}})/2 \underset{\to N}{\to} h_z^4 \; . \tag{10}$$

From Fig. 2 we see that h_x is opposed in sign to the coordinate z on the noon meridian contour. Thus, sufficiently close to N,

$$h_x = \pm\, h_z^2 \qquad \text{for } z \lessgtr 0 \; . \tag{11}$$

If we substitute into Eq. (7) and differentiate,

$$x'' = \pm\, dh_z/dz \; , \qquad z \lessgtr 0 \; , \tag{12}$$

which leaves two alternatives: either x" is discontinuous at N, or else x" = 0 at N. The latter implies c = 0, which is inconsistent with the geometry of the limiting field line in the xz-plane. Thus the curve, $\{x = x(z),\ y = 0\}$, must be represented as two separate power series for $z \lessgtr 0$. Since Eq. (11) must be satisfied on the noon meridian contour, it gives an implicit representation of that contour near N. This must agree with $x = \int dz\, h_x/h_z$ of Eq. (7) and with Fig. 2. The leading term of Eq. (11) is

$$2ax + nz^2 = \pm 4c^2z^2 \ . \tag{13}$$

If n = 0, Eq. (13) implies that a is of positive sign, which contradicts Fig. 2. Therefore, we must assume that n has two different values, according to the sign of z. This is permissible as long as $\underline{h}$ remains continuous at z = 0, and $\underline{\nabla}\cdot\underline{h} = 0$ everywhere. If we compare Eq. (13) with the leading term of Eq. (7) we find

$$x' = \pm h_z \leq 0, \ x = \pm cz^2 \tag{14}$$

near N. Thus, Eq. (13) becomes

$$n = \pm 2c(2c-a) \quad \text{for} \quad z \lessgtr 0 \ . \tag{15}$$

Surface Representation.

Since the gradient is in the x-direction at N, we can represent the surface by two second-order expansions in (y,z):

$$F(x,y,z) = x - A^{\pm}z^2 - B^{\pm}y^2 - C^{\pm}yz = 0 \tag{16}$$

for $z \lessgtr 0$. By symmetry, $B^+ = B^- = B$; Eq. (14) requires $A^+ = - A^- = c$. If we form the dot product, $\underline{H}\cdot\underline{\nabla}F = 0$, and substitute for x from Eq. (16), we find the confinement condition satisfied to second order in z^2, y^2, and yz when

$$m = 2B(2b - a) \ ; \tag{17}$$

$$C^{\pm}(1 - b/a - c/a) = 0 \ . \qquad (18)$$

The divergence conditions, Eqs. (6), require that d also be discontinuous, since $3d+m+n = 0$, although m is continuous. Therefore, from Eqs. (17), (15),

$$3d = \pm\ 2c(a - 2c) + 2B(a - 2b)\ ,\ z \lessgtr 0\ . \qquad (19)$$

Eq. (18) is incompatible with $a+b+c = 0$, unless $C^{\pm} = 0$. Although Eq. (19) forces h_x to be discontinuous at $z = 0$ ($a \neq 2c$), the $3dx^2$-term is negligibly small compared to the ax-term in Eq. (5).

Third-order terms in $\underline{H}\cdot\underline{\nabla}\ F = 0$ are eliminated by simply setting $f = 0 = s$. Then, by the divergence condition, $r = 0$, also. Thus, in the power series representation the confinement condition of Eq. (2) is satisfied up to fourth-order errors, on the postulated surface of Eq. (16). If we substitute the expansions for $\underline{h}$ and $F(x,y,z)$ into the pressure condition of Eq. (2) we find:

$$h_x^2 + h_y^2 + h_z^2 = \cos^2\chi \doteq (\partial F/\partial z)^2(1 - \text{2nd-order terms}), \qquad (20)$$

and expanding $\underline{h}$ yields

$$4c^2z^2 + 4b^2y^2 + \text{4th-order terms}$$
$$= 4c^2z^2\ (1 - \text{2nd-order terms})\ . \qquad (21)$$

By setting $b = 0$ we can reduce the error in the second boundary condition to fourth order. Our final expansion for the field in the neighborhood of N is given by:

$$-\Omega/\beta = cz^2 - cx^2 - (m\pm 6c^2)x^3/3 + mxy^2 \pm 6c^2xz^2 , \qquad (22)$$

and the surface of the magnetosphere near N is given by

$$F(x,y,z) = x - (\pm)cz^2 - my^2/2c , \qquad (23)$$

for $z \lessgtr 0$.

Multiple Reflections at N.

Midgeley and Davis (1963) have observed that the effect of multiple reflections of particles near N might seriously alter the pressure condition (Eq. (2)) near the neutral point. In the second-order approximation one can show that the shape of the surface is consistent with the pressure condition to within 4%. This effect would be most pronounced in the xz-plane; we shall calculate the added pressure, δp, at a point (x,z) on the noon meridian contour due to multiple reflections.

Figure 3 represents a particle incident on the point (x_o,z_o) on the contour $x = cz^2$, $z \leq 0$, $y = 0$, with an angle of incidence χ_o. This particle, on first reflection, strikes the magnetopause again at (x,z) where

$$(x-x_o)/(z-z_o) = c(z + z_o) = \tan 2\chi_o , \qquad (24)$$

and

$$\cot\chi_o = - x'(z_o) = - 2cz_o , \qquad (25)$$

therefore,

$$z = 4cz_o/(1 - 4c^2z_o^2) - z_o \doteq 3z_o \tag{26}$$

as indicated in Fig. 3. We note that the angle of incidence is $\pi - 2\chi_o + \chi$ at (x,z), and the change in the normal component of momentum will be correspondingly smaller. If we pursue the particle to the third point of reflection we find

$$c(z + 3z_o) = \tan 2(\chi-\chi_o) \ ; \ z \doteq 5z_o \ . \tag{27}$$

However, the effect of the particle, by this time, is negligible.

We now consider the additional pressure, δp, at (x,z) due to particles reflected from (x_o,z_o), and compare it with the pressure $p = 2nmV^2\cos^2\chi$ of the incident plasma stream (ignoring the y-coordinate). If $nVdx_o$ particles/sec are incident on an area of $ds_o = (dx_o^2 + dz_o^2)^{\frac{1}{2}}$ at (x_o,z_o) they are reflected onto an area $ds = (dx^2 + dz^2)^{\frac{1}{2}}$ at (x,z), where their momentum changes by $2mV\cos(\pi - 2\chi_o + \chi)$ per particle. Therefore, since $dx = 9\ dx_o$

$$\delta p = -2mnV^2\cos(\chi-2\chi_o)\,(dx/ds)/9 \ . \tag{28}$$

If we substitute for χ, χ_o, from Eq. (25) and note that $dx/ds = \cos\chi$, then we find:

$$\delta p/p = 1/27 \ll 1 \ . \tag{29}$$

We may conclude that the effect of multiple reflections near the neutral point is negligible, and that the pressure condition is valid over the entire magnetosphere, in this idealized representation.

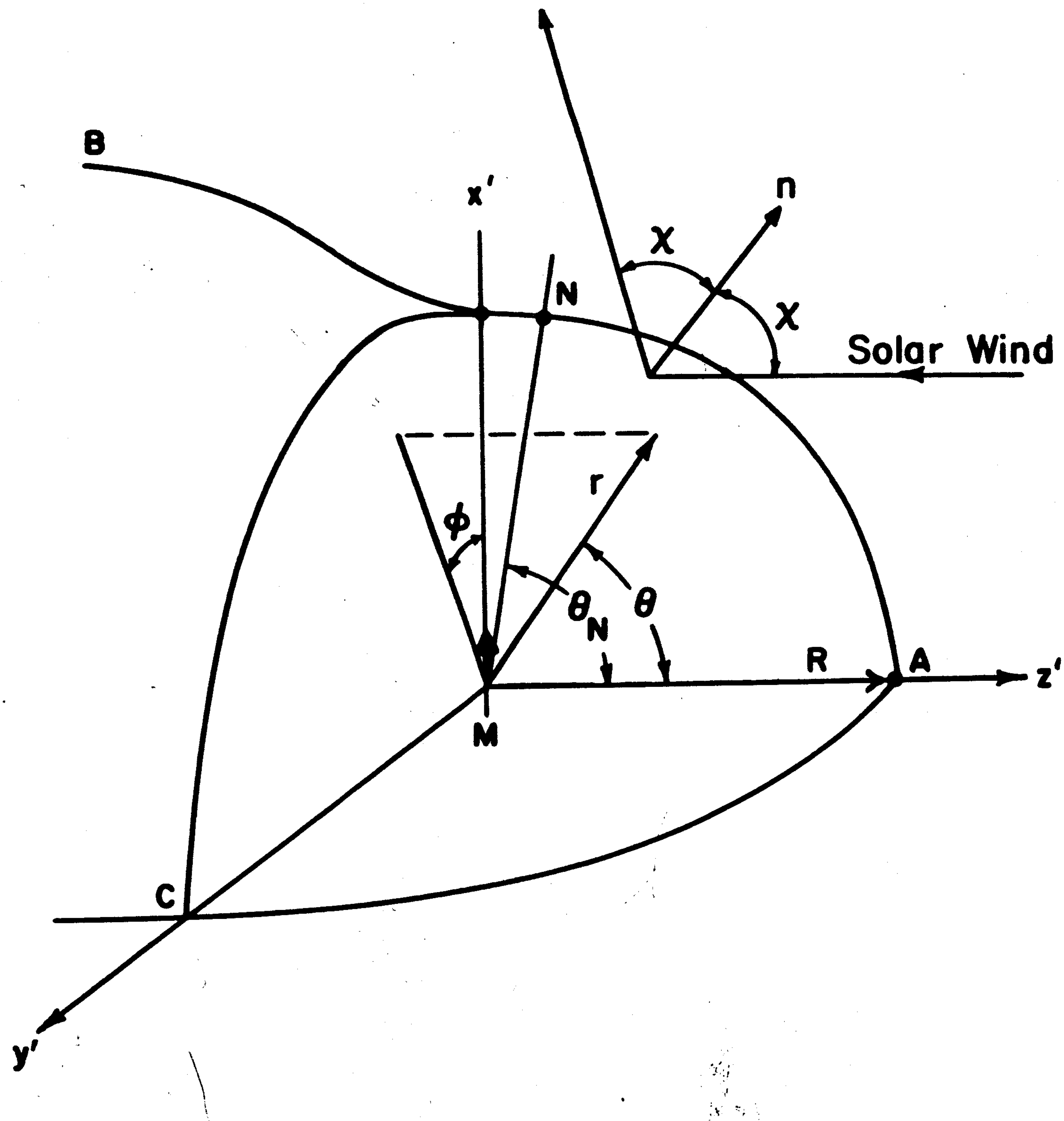

Fig. 1

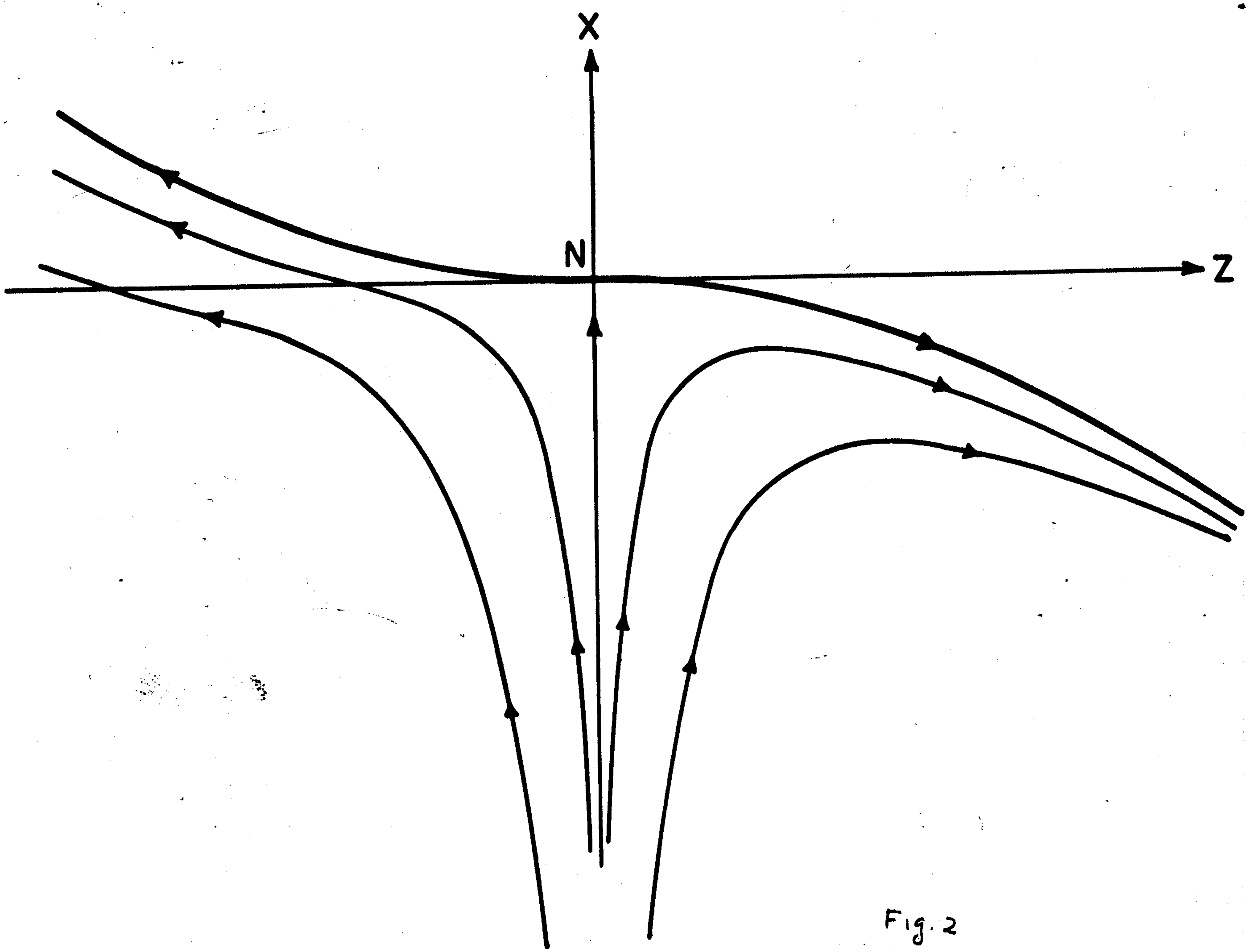

Fig. 2

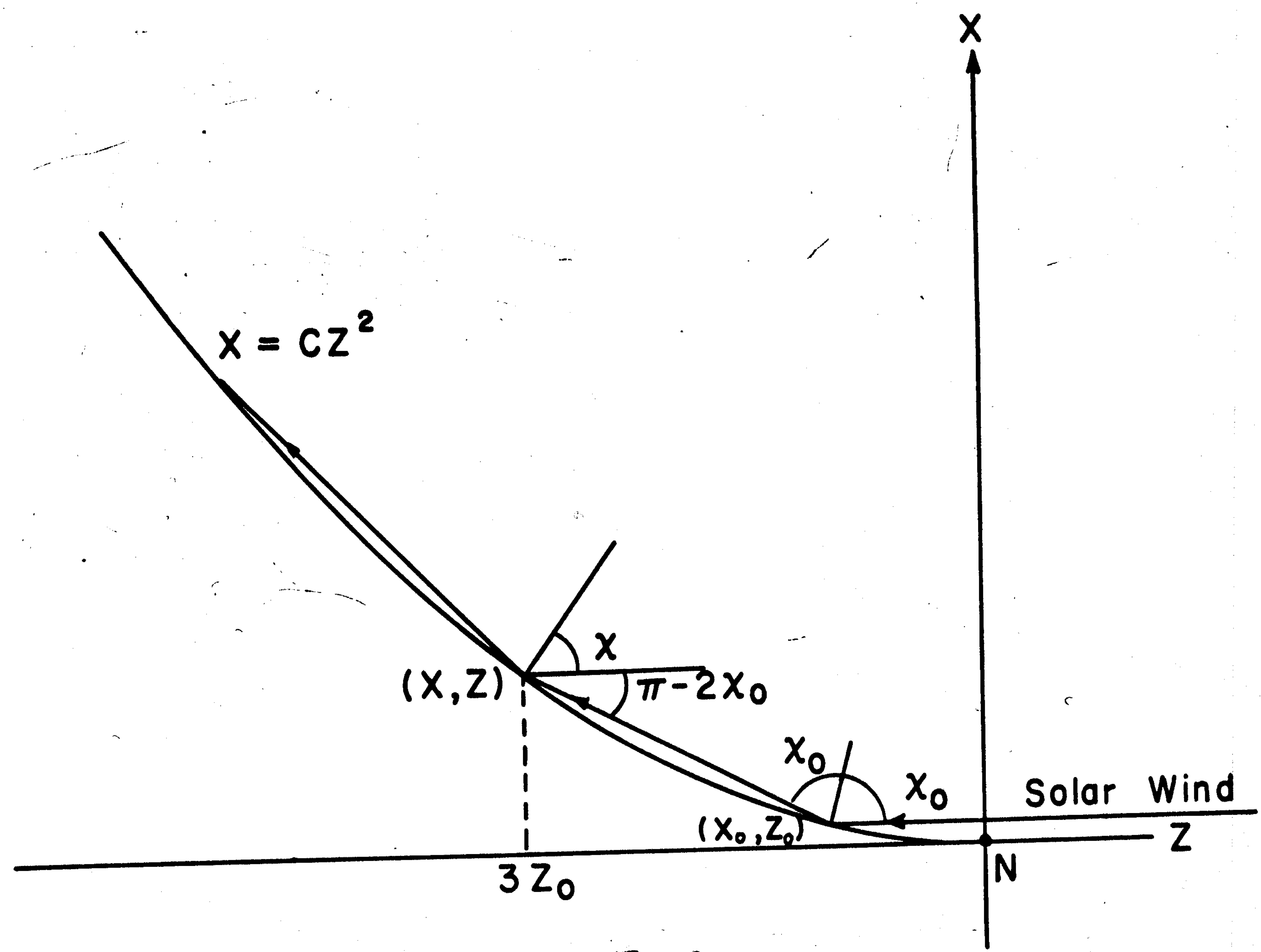

Fig. 3

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